Abstract
Standard Capital Asset Pricing Model (CAPM) fitting and testing using Quandl data.
CAPM Assumptions 1. Identical investors who are price takers; 2. Investment over the same time horizon; 3. No transaction costs or taxes; 4. Can borrow and lend at risk-free rate; 5. Investors only care about portfolio expected return and variance; 6. Market consists of all publicly traded assets.

The Consumption-Oriented CAPM is analogous to the simple form of the CAPM. Except that the growth rate of per capita consumption has replaced the rate of return on the market portfolio as the influence effecting returns.

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1 Fitting CAPM

1.1 Extracting and Organizing Data

```r
# 'Load the GARPFRM package and the CAPM dataset.
suppressMessages(library(GARPFRM))

## Warning: package 'PerformanceAnalytics' was built under R version 2.15.2

options(digits = 3)
data(capm_data)
stock.df <- capm_data
colnames(stock.df)

## [1] "Date" "MARKET" "WFC" "AAPL" "BP" "ATT" "RFREE" "CONS"

# Estimate a zooReg object: regularly spaced zoo object.
stock.z = zooReg(stock.df[, -1], start = c(1993, 1), end = c(2013, 11), frequency = 12)
index(stock.z) = as.yearmon(index(stock.z))

# Summarize Start, End, and Number of Rows
start(stock.z)

## [1] "Jan 1993"

dend(stock.z)

## [1] "Oct 2013"
nrow(stock.z)

## [1] 250
```

1.2 Estimate Excess Returns

Estimate excess returns: subtracting off risk-free rate. To strip off the dates and just return a
plain vector/matrix coredata() can be used.
# as.data.frame to check if an object is a data frame, or coerce it if possible.
returns.mat = as.matrix(coredata(stock.z))
exReturns.mat = returns.mat - returns.mat[, "RFREE"]
exReturns.df = as.data.frame(exReturns.mat)

### 1.3 Fitting CAPM Model

Run CAPM regression for AAPL (AAPL) using first 5 years (60 months divided by 12 months in a year = 5 years).

capm.fit = lm(AAPL ~ MARKET, data=exReturns.df, subset=1:60)
summary(capm.fit)

## Call:
## lm(formula = AAPL ~ MARKET, data = exReturns.df, subset = 1:60)
##
## Residuals:
##     Min       1Q   Median     3Q    Max
##-0.27680 -0.09200 -0.01290  0.07520  0.40230
##
## Coefficients:
##             Estimate Std. Error  t value  Pr(>|t|)
## (Intercept) -0.03840    0.02721  -1.4102   0.1603
## MARKET      0.52560    0.61910   0.8548   0.3991
##
## Residual standard error: 0.144 on 58 degrees of freedom
## Multiple R-squared: 0.01233, Adjusted R-squared: -0.00475
## F-statistic: 0.7211 on 1 and 58 DF,  p-value: 0.3991

# Plot data with regression line
plot(exReturns.df$MARKET, exReturns.df$AAPL, main="CAPM for AAPL",
     ylab="Excess Return: AAPL",
# Plot CAPM regression estimate
abline(capm.fit)

# Create Axis
abline(h=0,v=0,lty=3)

# Placing beta & tstat values on the plot for APPL
alpha = coef(summary(capm.fit))[1,1]
a_tstat = coef(summary(capm.fit))[1,3]
beta = coef(summary(capm.fit))[2,1]
b_tstat = coef(summary(capm.fit))[2,3]

legend("topleft", legend=
  c(paste("alpha =",round(alpha,dig=2),"","t =",round(a_tstat,dig=2),"")),
paste("beta =",round(beta,dig=2),"","t =",round(b_tstat,dig=2),"")), cex=1, bty="n")
2 Testing CAPM

2.1 Created CAPM Function

Use a capm.tstats function: Estimating CAPM with alpha=0 for asset using first 5 years of data

```r
capm.tstats = function(r, mkrt) {
  # Fitting CAPM
  capm.fit = lm(r ~ mkrt)
  # Extract summary info
  capm.summary = summary(capm.fit)
}
```
2.2 Estimate Significance and Test Beta Results

Retrieve tstats from function for assets. Filter out rf and market before running.

colnames(exReturns.mat[, -c(1, 6, 7)])
## [1] "WFC" "AAPL" "BP" "ATT"

tstats = apply(exReturns.mat[1:60, -c(1, 6, 7)], 2, capm.tstats, exReturns.mat[1:60, "MARKET"])
tstats
## WFC AAPL BP ATT
## 0.605 -1.413 -1.143 -1.779

# Test Hypothesis for 5% CI: H0: alpha=0
abs(tstats) > 2
## WFC AAPL BP ATT
## FALSE FALSE FALSE FALSE

any(abs(tstats) > 2)
## [1] FALSE

2.3 Estimate Expected Returns and Plot

Plot expected return versus beta. Estimate expected returns over first 5 years.

mu.hat = colMeans(exReturns.mat[1:60,-c(1,6,7)])
mu.hat
## WFC AAPL BP ATT
## -0.0421 -0.0552 -0.0317 -0.0394
# Compute beta over first 5 years

capm.betas = function(r, market) {
    capm.fit = lm(r ~ market)
    # Fit capm regression
    capm.beta = coef(capm.fit)[2]
    # Extract coefficients
    capm.beta
}

betas = apply(exReturns.mat[1:60, -c(1, 6, 7)], 2, FUN=capm.betas, market=exReturns.mat[1:60, "MARKET"])
betas
## WFC AAPL BP ATT
## 1.653 0.526 0.430 0.374

# Plot expected returns versus betas
plot(betas, mu.hat, main="Expected Return vs. Beta")
# Estimate regression of Expected Return vs. Beta

```r
sml.fit = lm(mu.hat ~ betas)
sml.fit
```

```r
## Call:
## lm(formula = mu.hat ~ betas)
##
## ## Coefficients:
## (Intercept) betas
## -0.04117 -0.00125
```
summary(sml.fit)

## Call:
## lm(formula = mu.hat ~ betas)
##
## Residuals:
## WFC AAPL BP ATT
## 0.00115 -0.01339 0.00997 0.00227
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -0.04117    0.01035  -3.98  0.058 .
## betas           -0.00125    0.01133  -0.11  0.922
##
## Residual standard error: 0.0119 on 2 degrees of freedom
## Multiple R-squared: 0.00607, Adjusted R-squared: -0.491
## F-statistic: 0.0122 on 1 and 2 DF, p-value: 0.922

# Ideally intercept is zero and equals the excess market return
mean(exReturns.mat[1:60,"MARKET"])

## [1] -0.032

# Plot Fitted SML
plot(betas,mu.hat,main="Estimated SML")
abline(sml.fit)
legend("topright",1, "Estimated SML",1)
3 Consumption-Oriented CAPM

3.1 Fitting C-CAPM

Run C-CAPM regression for CONS (Consumption) using first 5 years (60 months divided by 12 months in a year = 5 years).

```r
end = nrow(stock.z)
capm.fit = lm(CONS~MARKET,data=exReturns.df,(end-60):end)
summary(capm.fit)
```
## Call:
## lm(formula = CONS ~ MARKET, data = exReturns.df, subset = (end - 
## 60):end)

## Residuals:
## Min 1Q Median 3Q Max
## -1.8690 -0.1219 0.0334 0.1608 0.6851

## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.1189     0.0467  2.550  0.013 *
## MARKET        0.7245     0.9935  0.730  0.469

## Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

## Residual standard error: 0.356 on 59 degrees of freedom
## Multiple R-squared: 0.00893, Adjusted R-squared: -0.00787
## F-statistic: 0.532 on 1 and 59 DF, p-value: 0.469

# Plot data with regression line
plot(exReturns.df$MARKET, exReturns.df$CONS, main="CAPM for CONS",
     ylab="Excess Return: CONS",
     xlab="Excess Return: MARKET")

# Plot C-CAPM regression estimate
abline(capm.fit)

# Create Axis
abline(h=0,v=0, lty=3)

# Placing beta & tstat values on the plot for CONS
beta = coef(summary(capm.fit))[2,1]
b_stat = coef(summary(capm.fit))[2,3]
alpha = coef(summary(capm.fit))[1,1]
a_stat = coef(summary(capm.fit))[1,3]
NOTE: Specific problems with CCAPM is that it suffers from two puzzles: the equity premium puzzle (EPP) and the risk-free rate puzzle (RFRP). EPP implies that investors are extremely risk averse to explain the existence of a market risk premium. While RFRP stipulates that investors save in TBills despite the low rate of return.